Starter Qs – from last lesson

Q3 The random variable $X \sim B(20, 0.35)$. Find:

a) $P(X = 7)$ b) $P(X = 12)$ c) $P(2 < X \leq 4)$

d) $P(10 < X \leq 15)$ e) $P(7 \leq X \leq 10)$ f) $P(3 \leq X < 11)$

Q4 The random variable $X \sim B(25, 0.8)$. Find:

a) $P(X \geq 17)$ b) $P(X \geq 20)$ c) $P(X > 14)$

d) $P(X = 21)$ e) $P(14 \leq X < 17)$ f) $P(12 < X \leq 18)$

Q5 The probability of having green eyes is known to be 0.18. In a class of thirty children, find the probability that fewer than ten children have green eyes.

Q6 In a production process it is known that approximately 5% of items are faulty. In a random sample of 25 objects, estimate the probability that fewer than 6 are faulty.

N Statistical distributions

N1	Understand and use simple, discrete probability distributions (calculation of mean and variance of discrete random variables is excluded), including the binomial distribution, as a model; calculate probabilities using the binomial distribution.

N Statistical distributions

Students should:

- recognise when a situation may be modelled by a discrete random variable
- know and be able to use the fact that the sum of the probabilities of all possible outcomes of an event is 1
- understand a discrete probability distribution defined in a table or by a function
- be able to find the probability of a defined event in a given context
- recognise and be able to use $\mathrm{B}(n, p)$ as the notation for a binomial distribution with n independent trials where p is the probability of 'success' at any trial
- be able to state the conditions necessary for a binomial distribution and assess whether they are likely to be valid in a given situation
- be able to find the probability of an exact number of successes in a binomial distribution using the formula (given in the formulae book) or on a calculator
- be able to find cumulative probabilities in a binomial distribution
- calculate the mean, variance and standard deviation of a binomial distribution using the standard formulae given in the formulae book
- be able to use $\mathrm{P}(X \geq n) = 1 - \mathrm{P}(X \leq n-1)$ and similar results

Note: when using the binomial distribution, students must use a calculator to find probabilities.

10.2 Binomial Distribution

Example 1: *Question 6 from Exercise 10.2A*

A bag contains 12 counters. Three are red and the rest are black. A sample of five counters is taken, placing each back in the bag after it is chosen. Find the probability that the sample contains more than three red counters.

Check the conditions for binomial distribution are met:

1. *Independent*
2. *Fixed number of trials*
3. *Fixed probability*

10.2 Binomial Distribution

Example 1: *Question 6 from Exercise 10.2A*

A bag contains 12 counters. Three are red and the rest are black. A sample of five counters is taken, placing each back in the bag after it is chosen. Find the probability that the sample contains more than three red counters.

Identify the random variable and the corresponding values of n and p:

Let be the number of red counters in the sample.

 = number of trials = 5, = probability of

10.2 Binomial Distribution

Example 1: *Question 6 from Exercise 10.2A*

A bag contains 12 counters. Three are red and the rest are black. A sample of five counters is taken, placing each back in the bag after it is chosen. Find the probability that the sample contains more than three red counters.

Calculate the probabilities:

10.2 Binomial Distribution

Example 2

A bag of sweets contains fudges, toffees and caramels in the ratio Twelve sweets are selected from the bag with replacement. Stating any assumptions you make, calculate the probability of:

a) Four fudges being chosen

Let be the number of fudges. Assume the selections are independent of each other and that each sweet has an equal chance of being chosen.

10.2 Binomial Distribution

Example 2

A bag of sweets contains fudges, toffees and caramels in the ratio Twelve sweets are selected from the bag with replacement. Stating any assumptions you make, calculate the probability of:

b) At least six sweets in the sample being either toffees or caramels.

Let be the number of toffees or caramels.

= P(toffee or caramel) = 0.3 + 0.3 = 0.6

10.2 Binomial Distribution

Example 3

Ten students take a test. They all have a probability p of passing, independent of the results of other students. X is the number of students passing the test.

If $P(X = 6) = 4 \times P(X = 4)$, find the value of p.

$$P(X=6)=\binom{10}{6}p^6(1-p)^4=210\,p^6(1-p)^4$$

$$P(X=4)=\binom{10}{4}p^4(1-p)^6=210\,p^4(1-p)^6$$

$$\therefore 210\,p^6(1-p)^4=840\,p^4(1-p)^6$$

We don't know so cannot use

10.2 Binomial Distribution

Example 3

Ten students take a test. They all have a probability p of passing, independent of the results of other students. X is the number of students passing the test.

If $P(X = 6) = 4 \times P(X = 4)$, find the value of p.

$$\therefore 210p^6(1-p)^4 = 840p^4(1-p)^6$$

(probabilities are < 1)

10.2 Binomial Distribution

Example 4i

Extensive research has shown that 1 person out of every 4 is allergic to a particular grass pollen.

A group of 20 university students volunteer to try out a new treatment.

(i) What is the expectation of the number of allergic people in the group?

is the probability that the person is allergic, .
is the number of students in the group, .
Let be the number of students in the sample that are allergic.

Note that np is the mean. You also need to be able to find the standard

(i) Expectation = **people**.

10.2 Binomial Distribution

Example 4ii

(ii) What is the probability that

(a) exactly two (b) no more than two of the group are allergic?

(iii) How large a sample would be needed for the probability of it containing at least one allergic person to be greater than 99.9%?

(iv) What assumptions have you made in your answer?

(ii)(a)

(b)

10.2 Binomial Distribution

Example 4iii

(ii) What is the probability that

(a) exactly two (b) no more than two of the group are allergic?

(iii) How large a sample would be needed for the probability of it containing at least one allergic person to be greater than 99.9%?

(iv) What assumptions have you made in your answer?

$X \sim B(n, 0.25)$

(iii) We don't know , so cannot use the calculator to find … so use the formula…

So we need:

25 people required

10.2 Binomial Distribution

Example 4iv

(ii) What is the probability that

(a) exactly two (b) no more than two of the group are allergic?

(iii) How large a sample would be needed for the probability of it containing at least one allergic person to be greater than 99.9%?

(iv) What assumptions have you made in your answer?

(iv) (a) That the sample is random. This is likely to be untrue as most university students are aged 18-25 so a sample of them cannot realistically be a random sample of the whole population.

(b) That the outcome for one person is